

State after quantum tunneling with gravity

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Abstract

The Wheeler-DeWitt equation is investigated and used to examine a state after a quantum tunneling with gravity. To make arguments definite we treat a discretized version of the Wheeler-DeWitt equation and adopt the WKB method. We expand an Euclidean wave function around an instanton, by using a deviation equation of a vector field tangent to a congruence of instantons. The instanton around which we expand the wave function corresponds to a so-called most probable escape path (MPEP). It is shown that, when the wave function is analytically continued, the corresponding state of physical perturbations is equivalent to the vacuum state determined by positive-frequency mode functions which satisfy appropriate boundary conditions. Thus a quantum field theory is effective to investigate a state after a quantum tunneling with gravity. The effective Lagrangian describing the field theory is obtained by simply reducing the original Lagrangian to a subspace spanned by the physical perturbations. The result of this paper does not depend on the operator ordering and can be applied to all physical perturbations, including gravitational perturbations, around a general MPEP.

I. INTRODUCTION

The universe is thought to have been experienced phase transitions many times. Some of these phase transitions may be due to quantum tunneling. Hence there arises an intellectual and practical urge to know a state after a quantum tunneling. For this purpose many authors investigated the Schrodinger equation of scalar fields in flat spacetime [1,2] in the multi-dimensional tunneling approach [3]. They showed that a state after a quantum tunneling in flat spacetime is the vacuum state determined by positive-frequency mode functions which satisfy appropriate boundary conditions. The result says that a quantum field theory of the scalar fields is effective to seek a particle spectrum after a quantum tunneling if the boundary conditions are attached on the mode functions. The effectiveness makes it possible to calculate a CMB anisotropy of the universe theoretically in some inflationary scenarios [4].

However the calculated anisotropy is based on the formalism developed in flat spacetime, as mentioned above. Since gravitational effects can be significant in some cases [5], we want to generalize the formalism to include gravity. As far as only the scalar fields' perturbations are concerned, the generalization was done in Ref. [6], by neglecting gravitational perturbations. In this case, a state of the scalar fields' perturbations after a quantum tunneling with gravity is obtained by the same method as in flat spacetime, except evolution equations of mode functions is replaced by those on the curved background. Thus a quantum field theory of the scalar fields in curved spacetime is effective to investigate a state of the scalar fields' perturbations after a quantum tunneling, provided that gravitational perturbations are neglected. However, when we intend to investigate a state of gravitational perturbations after a quantum tunneling (for example, when we calculate a spectrum of the primordial gravitational waves), we have to include gravitational perturbations. The corresponding formalism must be derived from a more fundamental level.

When we investigate a classical dynamics of gravitational perturbations, so-called gauge invariant variables play a important role. The gauge invariant variables are those linear combinations of perturbations which are invariant to linear order under coordinate trans-

formations, and a maximal set of them describes all physical degrees of freedom of linear perturbations. For several background geometries, it was shown that classical dynamics of the gauge invariant variables can be described by some auxiliary scalar fields (Ref. [7] for Minkowski background, Ref. [8] for the Robertson-Walker background, Ref. [9] for the Milne background, and Ref. [10] for the one-bubble inflationary background). Hence we can expect that a quantum field theory of the auxiliary scalar fields can be applied to investigate a state of gravitational perturbations after a quantum tunneling, although the equivalence is established only in a classical level.

At this point, we can expect that a quantum field theory will be effective to investigate a state of all physical perturbations after a quantum tunneling with gravity. Thus we want to show from a quantum theory of gravity that the expectation is true. In all papers with this aim [11], the Wheeler-DeWitt equation is expanded by some collective coordinates around a background. Our strategy in this paper is to expand not the Wheeler-DeWitt equation but a wave function itself covariantly. Anyway, due to the result of this paper, we can safely use the quantum field theory of perturbations in order to calculate a CMB anisotropy, a spectrum of the primordial gravitational waves, etc. in some inflationary scenarios.

Now let us review a quantum theory of gravity based on a canonical formalism. Consider a system of a n -dimensional spacetime $(M, g_{\mu\nu})$ without boundary and scalar fields χ^a described by the action

$$I_{E-KG} = \int d^n x \sqrt{-g} \left[\frac{1}{2\kappa^2} R - \frac{1}{2} g^{\mu\nu} \gamma_{ab}(\chi^c) \partial_\mu \chi^a \partial_\nu \chi^b - V(\chi^a) \right] , \quad (1.1)$$

where γ_{ab} is an arbitrary positive-definite matrix. In the ADM decomposition [7] of the spacetime

$$ds^2 = -\Lambda_\perp^2 dt^2 + q_{ij}(dx^i + \Lambda^i dt)(dx^j + \Lambda^j dt) , \quad (1.2)$$

functional derivatives of the action I_{E-KG} with respect to Λ_\perp and Λ^i give the following constraints.

$$\mathcal{H}_{\perp\mathbf{x}} = 0 ,$$

$$\mathcal{H}_i \mathbf{x} = 0 \quad , \quad (1.3)$$

where

$$\begin{aligned} \mathcal{H}_\perp \mathbf{x} &\equiv \kappa^2 G_{ijkl} \pi^{ij} \pi^{kl} + \frac{1}{2\sqrt{q}} \gamma^{ab} \pi_a \pi_b \\ &\quad + \sqrt{q} \left[-\frac{(n-1)R}{2\kappa^2} + \frac{1}{2} q^{ij} \gamma_{ab}(\chi^c) \partial_i \chi^a \partial_j \chi^b + V(\chi^a) \right] \Big|_{\mathbf{x}} \quad , \\ \mathcal{H}_i \mathbf{x} &\equiv -2\sqrt{q} D_j \left(\frac{\pi_i^j}{\sqrt{q}} \right) + \pi_a \partial_i \chi^a \Big|_{\mathbf{x}} \quad . \end{aligned} \quad (1.4)$$

The former is called Hamiltonian constraints and the later is called momentum constraints. In the expressions, π^{ij} is a momentum conjugate to q_{ij} , π_a is one conjugate to χ^a , D_j denotes a covariant derivative compatible with q_{ij} , and

$$G_{ijkl} \equiv \frac{1}{\sqrt{q}} \left(q_{ik} q_{jl} + q_{il} q_{jk} - \frac{2}{n-2} q_{ij} q_{kl} \right) \quad (1.5)$$

for $n \neq 2$,

$$G_{ijkl} \equiv \frac{1}{\sqrt{q}} (q_{ik} q_{jl} + q_{il} q_{jk}) \quad (1.6)$$

for $n = 2$. It is well-known that Poisson brackets among the above constraints are given by

$$\begin{aligned} \{f\mathcal{H}_\perp, g\mathcal{H}_\perp\}_P &= (fD^i g - gD^i f) \mathcal{H}_i \quad , \\ \{f^i \mathcal{H}_i, g\mathcal{H}_\perp\}_P &= (f^i \partial_i g) \mathcal{H}_\perp \quad , \\ \{f^i \mathcal{H}_i, g^j \mathcal{H}_j\}_P &= (f^j D_j g^i - g^j D_j f^i) \mathcal{H}_i \quad , \end{aligned} \quad (1.7)$$

where $f\mathcal{H}_\perp$ and $f^i \mathcal{H}_i$ denote $\int d\mathbf{x} f(\mathbf{x}) \mathcal{H}_\perp \mathbf{x}$ and $\int d\mathbf{x} f^i(\mathbf{x}) \mathcal{H}_i \mathbf{x}$, respectively. These show that the constraints are first class. A method to quantize a system with first class constraints was given by Dirac [12] and leads the following simultaneous differential equations for a wave functional Ψ .

$$\begin{aligned} \hat{\mathcal{H}}_\perp \mathbf{x} \Psi &= 0 \quad , \\ \hat{\mathcal{H}}_i \mathbf{x} \Psi &= 0 \quad , \end{aligned} \quad (1.8)$$

where $\hat{\mathcal{H}}_{\perp\mathbf{x}}$ and $\hat{\mathcal{H}}_{i\mathbf{x}}$ are differential operators obtained by replacing π^{ij} with $-i\hbar\delta/\delta q_{ij}$, π_a with $-i\hbar\delta/\delta\chi^a$ in $\mathcal{H}_{\perp\mathbf{x}}$ and $\mathcal{H}_{i\mathbf{x}}$, respectively. Equation (1.8) is called the Wheeler-DeWitt equation [13]. In defining the differential operators, there is a problem of operator ordering. The first principle to determine the operator ordering is requiring the following algebra [14]:

$$\left[f\hat{\mathcal{H}}_{\perp}, g\hat{\mathcal{H}}_{\perp}\right]\Psi = i\hbar(fD^ig - gD^if)\hat{\mathcal{H}}_i\Psi \quad , \quad (1.9)$$

$$\left[f^i\hat{\mathcal{H}}_i, g\hat{\mathcal{H}}_{\perp}\right]\Psi = i\hbar(f^i\partial_ig)\hat{\mathcal{H}}_{\perp}\Psi \quad , \quad (1.10)$$

$$\left[f^i\hat{\mathcal{H}}_i, g^j\hat{\mathcal{H}}_j\right]\Psi = i\hbar[f, g]^i\hat{\mathcal{H}}_i\Psi \quad (1.11)$$

for an arbitrary functional Ψ . The algebra is a quantum version of (1.7), and is called the Dirac algebra [12]. The condition (1.9-1.11) is necessary in order for the equation (1.8) to give a consistent quantum theory.

The remaining part of this paper is organized as follows. In Sec. II we investigate a simultaneous differential equations, which become the Wheeler-DeWitt equation in a limit. We expand an Euclidean wave function around an instanton, by using a deviation equation of a vector field tangent to a congruence of instantons. The instanton around which we expand the wave function corresponds to a so-called most probable escape path (MPEP). In Sec. III, by analytically continuing the wave function, it is shown that the corresponding state of physical perturbations after a quantum tunneling is the vacuum state determined by positive-frequency mode functions which satisfy appropriate boundary conditions. Hence a quantum field theory is effective to investigate a state after a quantum tunneling. Sec. IV is devoted to summarize this paper.

II. EUCLIDEAN WAVE FUNCTION FOR A DISCRETIZED WHEELER-DEWITT EQUATION

Let $(\mathcal{M}_x, G_{x\alpha\beta})$ be a family of D -dimensional pseudo-Riemannian manifolds parameterized by an integer x ($= 1, 2, \dots, x_{max}$). A configuration space we consider is a pseudo-Riemannian manifold constructed from them as

$$(\mathcal{M}, G_{\alpha\beta}) = \oplus_x (\mathcal{M}_x, G_{x\alpha\beta}) \quad . \quad (2.1)$$

Then we consider the following simultaneous differential equations for a complex function Ψ on \mathcal{M} .

$$\begin{aligned} \hat{\mathcal{H}}_{\perp x} \Psi &= 0 \quad , \\ \hat{\mathcal{H}}_I \Psi &= 0 \quad , \end{aligned} \quad (2.2)$$

where $x = 1, 2, \dots, x_{max}$, $I = 1, 2, \dots, I_{max}$ and

$$\begin{aligned} \hat{\mathcal{H}}_{\perp x} &\equiv -\frac{\hbar^2}{2} G_x^{\alpha\beta} D_\alpha D_\beta + V_x \quad , \\ \hat{\mathcal{H}}_I &\equiv -i\hbar v_I^\alpha D_\alpha \end{aligned} \quad (2.3)$$

are linear differential operators on \mathcal{M} . As usual we call a solution of this equation a *wave function*. In the expression D is a covariant derivative compatible with the metric $G_{\alpha\beta}$, $\{V_x\}$ is a set of functions on \mathcal{M} , and $\{v_I^\alpha\}$ is a set of linearly independent vectors on \mathcal{M} . The Wheeler-DeWitt equation (1.8) can be written in this form provided that it is properly discretized and the operator ordering problem is ignored. $\hat{\mathcal{H}}_{\perp x}$ corresponds to the Hamiltonian constraint at the point x on a spacelike hypersurface, and $\hat{\mathcal{H}}_I$ corresponds to the momentum constraint for a point and a direction both of which are specified by I . For example,

$$\begin{aligned} G_x^{q_{ij}(x')q_{kl}(x'')} &= 2\kappa^2 G_{ijkl}(x) \delta_{xx'} \delta_{xx''} \quad , \\ G_x^{q_{ij}(x')\chi^a(x'')} &= 0 \quad , \\ G_x^{\chi^a(x')\chi^b(x'')} &= \frac{1}{\sqrt{q(x)}} \gamma^{ab}(x) \delta_{xx'} \delta_{xx''} \end{aligned} \quad (2.4)$$

for the system considered in Sec. I, where G_{ijkl} is defined by (1.6). We mention that terms linear in the derivative, which may appear when the operator ordering problem is solved, were not included in $\mathcal{H}_{\perp x}$ for concreteness, since they do not change our conclusion.

For the equations to be solved consistently it must be assumed that any commutators between the linear operators can be written as linear combinations of themselves when they

operate on an arbitrary function on the configuration space. Equivalently they must generate a Lie algebra with respect to the commutators. In this paper we adopt the following algebra of commutators between the differential operators.

$$[\hat{\mathcal{H}}_{\perp x}, \hat{\mathcal{H}}_{\perp x'}] \Psi = i\hbar c_{xx'}^{(1)I} \hat{\mathcal{H}}_I \Psi \quad , \quad (2.5)$$

$$[\hat{\mathcal{H}}_I, \hat{\mathcal{H}}_{\perp x}] \Psi = i\hbar c_{Ix}^{(2)x'} \hat{\mathcal{H}}_{\perp x'} \Psi \quad , \quad (2.6)$$

$$[\hat{\mathcal{H}}_I, \hat{\mathcal{H}}_J] \Psi = i\hbar c_{IJ}^{(3)K} \hat{\mathcal{H}}_K \Psi \quad (2.7)$$

for an arbitrary function Ψ on \mathcal{M} . Note that the right hand side of (1.9), (1.10) and (1.11) are linear in $\hat{\mathcal{H}}_i \Psi$'s, $\hat{\mathcal{H}}_{\perp} \Psi$'s and $\hat{\mathcal{H}}_i \Psi$'s, respectively while the right hand side of (2.5), (2.6) and (2.7) are linear in $\hat{\mathcal{H}}_I \Psi$'s, $\hat{\mathcal{H}}_{\perp x} \Psi$'s and $\hat{\mathcal{H}}_I \Psi$'s, respectively. Hence the algebra (2.5-2.7) is a generalization of the discretized version of the Dirac algebra (1.9-1.11). We include the case when the 'structure constants' $c^{(1)}$, $c^{(2)}$ and $c^{(3)}$ depend on a position in the configuration space \mathcal{M} . Although confirmation of the algebra requires a knowledge of short distance behaviors of the theory, we simply assume that the algebra does hold. After detailed investigation of the discretized system, we will extract results independent of not only the operator ordering but also a way of the discretization. Note that since a discretized version of the Dirac algebra (1.9-1.11) can be written in the form (2.5-2.7), the system we consider includes the discretized Wheeler-DeWitt equation as an important example. Now, for later conveniences we rewrite the assumption (2.5-2.7) as follows. It is equivalent to the following set of equalities

$$G_x^{\alpha\beta} \partial_{\beta} V_{x'} - G_{x'}^{\alpha\beta} \partial_{\beta} V_x = -c_{xx'}^{(1)I} v_I^{\alpha} \quad , \quad (2.8)$$

$$G_x^{\alpha\gamma} D_{\gamma} v_I^{\beta} + G_x^{\beta\gamma} D_{\gamma} v_I^{\alpha} = c_{Ix}^{(2)x'} G_{x'}^{\alpha\beta} \quad , \quad (2.9)$$

$$v_I^{\alpha} \partial_{\alpha} V_x = -c_{Ix}^{(2)x'} V_{x'} \quad , \quad (2.10)$$

$$[v_I, v_J]^{\alpha} = -c_{IJ}^{(3)K} v_K^{\alpha} \quad (2.11)$$

and

$$G_x^{\alpha\beta} D_{\alpha} D_{\beta} V_{x'} - G_{x'}^{\alpha\beta} D_{\alpha} D_{\beta} V_x = 0 \quad , \quad (2.12)$$

$$G_x^{\alpha\beta} D_{\alpha} D_{\beta} v_I^{\gamma} = -G_x^{\alpha\beta} v_I^{\delta} R_{\alpha\delta\beta}^{\gamma} \quad , \quad (2.13)$$

where R is a curvature tensor of D :

$$R(X, Y)Z = D_X D_Y Z - D_Y D_X Z - D_{[X, Y]}Z \quad . \quad (2.14)$$

Note that, while (2.12-2.13) are affected by the operator ordering, (2.8-2.11) are not. In particular the fact that $\{v_I^\alpha\}$ is integrable is independent of the operator ordering since it is a consequence of (2.11). We denote the integral surface G . In this paper the quotient space \mathcal{M}/G plays an important role since the second equation of (2.2) shows that G has nothing to do with physical degrees of freedom. In the case of the usual Wheeler-DeWitt equation the quotient space \mathcal{M}/G is called a superspace. Hence we call the quotient space a *superspace* in our case, too. Evidently, the superspace is a space of all physically distinct configurations.

Next we investigate a classical mechanical system whose quantum version corresponds to the equations (2.2). Let us consider a phase space $(T^*\mathcal{M}, dp_\alpha \wedge dq^\alpha)$ and a Hamiltonian of the form ¹

$$H = \Lambda_\perp^x \mathcal{H}_{\perp x} + \Lambda^I \mathcal{H}_I \quad , \quad (2.15)$$

where Λ_\perp^x ($\neq 0$) and Λ^I are Lagrange's multipliers and

$$\begin{aligned} \mathcal{H}_{\perp x} &\equiv \frac{1}{2} G_x^{\alpha\beta} p_\alpha p_\beta + V_x \quad , \\ \mathcal{H}_I &\equiv v_I^\alpha p_\alpha. \end{aligned} \quad (2.16)$$

In the expression, p_α ($\in T^*\mathcal{M}$) is a momentum conjugate to a coordinate q^α of \mathcal{M} . (2.8-2.11) is equivalent to the following algebra of Poisson brackets.

$$\begin{aligned} \{\mathcal{H}_{\perp x}, \mathcal{H}_{\perp x'}\}_P &= c_{xx'}^{(1)I} \mathcal{H}_I \quad , \\ \{\mathcal{H}_I, \mathcal{H}_{\perp x}\}_P &= c_{Ix}^{(2)x'} \mathcal{H}_{\perp x'} \quad , \\ \{\mathcal{H}_I, \mathcal{H}_J\}_P &= c_{IJ}^{(3)K} \mathcal{H}_K \quad . \end{aligned} \quad (2.17)$$

¹ Hereafter we apply the Einstein's summation rule unless otherwise stated.

The equivalence is consistent with the well-known fact that the operator ordering does not affect the corresponding classical dynamics. The classical equations of motion in the configuration space \mathcal{M} is obtained from the constraint equations and the Hamilton's equation as follows ².

$$\begin{aligned}\frac{1}{2}G_{x\alpha\beta}\tilde{\mathcal{N}}^\alpha\tilde{\mathcal{N}}^\beta + (\Lambda_\perp^x)^2 V_x &= 0 \quad , \\ v_I^\alpha \mathcal{G}_{\alpha\beta}\tilde{\mathcal{N}}^\beta &= 0 \quad , \\ \tilde{\mathcal{N}}^\beta \mathcal{D}_\beta \tilde{\mathcal{N}}^\alpha &= -\mathcal{G}^{\alpha\beta}\partial_\beta \mathcal{V} + \mathcal{G}^{\alpha\beta}\mathbf{v}^\gamma (\partial_\beta w_\gamma - \partial_\gamma w_\beta) \quad ,\end{aligned}\tag{2.18}$$

where

$$\begin{aligned}\tilde{\mathcal{N}}^\alpha &\equiv \frac{dq^\alpha}{dt} - \mathbf{v}^\alpha \quad , \\ \mathbf{v}^\alpha &\equiv \Lambda^I v_I^\alpha \quad , \\ \mathcal{G}^{\alpha\beta} &\equiv \Lambda_\perp^x G_x^{\alpha\beta} \quad , \\ \mathcal{G}_{\alpha\beta} &\equiv (\mathcal{G}^{-1})_{\alpha\beta} \quad , \\ \mathcal{V} &\equiv \Lambda_\perp^x V_x \quad ,\end{aligned}\tag{2.19}$$

and

$$w_\alpha \equiv \mathcal{G}_{\alpha\beta}\tilde{\mathcal{N}}^\beta \quad .\tag{2.20}$$

In the expression \mathcal{D} is a covariant derivative compatible with the metric $\mathcal{G}_{\alpha\beta}$.

Now we return to the problem of solving the simultaneous differential equations (2.2) to obtain a wave function. To extract those properties of the equations which are independent of the operator ordering we adopt the WKB method. First, without loss of generality, we can expand the wave function Ψ as

$$\Psi = \exp \left[-\frac{1}{\hbar}(W^{(0)} + \hbar W^{(1)} + \dots) \right] \quad ,\tag{2.21}$$

where $W^{(0)}, W^{(1)}, \dots$ are complex functions on \mathcal{M} . Next we solve the differential equations order by order in \hbar , considering \hbar as a small parameter.

² Here the Einstein's summation rule is not applied with respect to x .

A. Lowest-order WKB wave function

To the lowest order in \hbar the differential equation (2.2) is reduced to the following simultaneous differential equations for $W^{(0)}$.

$$\begin{aligned}\frac{1}{2}G_x^{\alpha\beta}\partial_\alpha W^{(0)}\partial_\beta W^{(0)} &= V_x \quad , \\ v_I^\alpha\partial_\alpha W^{(0)} &= 0 \quad .\end{aligned}\tag{2.22}$$

These determines $W^{(0)}$ as a function on \mathcal{M} , provided that a suitable boundary condition is attached. When $W^{(0)}$ is real in a region of the configuration space, we call the corresponding wave function Ψ an *Euclidean wave function*. We call the region an *Euclidean region*. In the remaining of this paper we investigate the Euclidean wave function. Physical meaning of the Euclidean wave function will become clear in the following arguments.

For the real function $W^{(0)}$ we can define a family of vector fields $\{N_x^\alpha\}$ parameterized by the integer x :

$$N_x^\alpha \equiv G_x^{\alpha\beta}\partial_\beta W^{(0)}.\tag{2.23}$$

Operating the differential operator $G_{x'}^{\alpha\beta}D_\beta$ on the first equation of (2.22), we obtain the following equation.

$$N_x^\beta D_\beta N_{x'}^\alpha = G_{x'}^{\alpha\beta}\partial_\beta V_x \quad ,\tag{2.24}$$

where we have used the fact that the covariant derivative D is torsion free and compatible with $G_{x\alpha\beta}$. Note that the compatibility with $G_{x\alpha\beta}$ is a result of the direct-sum structure (2.1) of $(\mathcal{M}, G_{\alpha\beta})$.

Since the first of (2.22) is of the form of the Hamilton-Jacobi equation, we may regard (2.24) as the corresponding equations of motion for the system. In fact, if we define a vector field \mathcal{N}^α by

$$\mathcal{N}^\alpha \equiv \Lambda_\perp^x N_x^\alpha \quad ,\tag{2.25}$$

then we can show that

$$\begin{aligned}
\frac{1}{2}G_{x\alpha\beta}\mathcal{N}^\alpha\mathcal{N}^\beta - (\Lambda_\perp^x)^2 V_x &= 0 \quad , \\
v_I^\alpha \mathcal{G}_{\alpha\beta}\mathcal{N}^\beta &= 0 \quad , \\
\mathcal{N}^\beta \mathcal{D}_\beta \mathcal{N}^\alpha &= \mathcal{G}^{\alpha\beta} \partial_\beta \mathcal{V} \quad .
\end{aligned} \tag{2.26}$$

Since $\partial_\beta \omega_\gamma - \partial_\gamma \omega_\beta = 0$ in this case, these equations show that \mathcal{N}^α satisfies the classical equations of motion (2.18) in which V_x is replaced by $-V_x$, where $\omega_\alpha \equiv \mathcal{G}_{\alpha\beta}\mathcal{N}^\beta$. Therefore \mathcal{N}^α defined by (2.25) is a tangent vector field of a congruence of instantons. In this sense the Euclidean wave function define a congruence of instantons. Conversely, if a congruence of instantons satisfies the condition $\boldsymbol{v}^\gamma(\partial_\beta w_\gamma - \partial_\gamma w_\beta) = 0$, then we can construct the corresponding lowest-order Euclidean wave function as follows. Introduce a parameter τ by

$$\left(\frac{\partial}{\partial\tau}\right)^\alpha = \tilde{\mathcal{N}}^\alpha \quad , \tag{2.27}$$

and

$$\frac{\partial W^{(0)}}{\partial\tau} = 2\mathcal{V} \quad . \tag{2.28}$$

B. Expansion of the Euclidean wave function around an instanton

In this subsection we consider a real solution of (2.22) and expand it around an instanton. An expansion of the corresponding Euclidean wave function is obtained from that.

The second of the constraint equations (2.2) or the second of (2.22) suggests that any equations for physical degrees of freedom can be written in forms invariant under any diffeomorphism of G . Hence we can expect that there is a set of real functions $\{\lambda_\perp^x\}$ such that a weighted metric $\lambda_\perp^x G_x^{\alpha\beta}$ and a weighted 'potential' $\lambda_\perp^x V_x$ are invariant under the diffeomorphism. We first investigate the weighted metric. With the help of (2.9) it can be shown that the condition $L_{v_I}(\lambda_\perp^x G_x^{\alpha\beta}) = 0$ is equivalent to

$$v_I^\alpha \partial_\alpha \lambda_\perp^x = \lambda_\perp^{x'} c_{Ix'}^{(2)x} \quad , \tag{2.29}$$

where \mathbf{L} represents a Lie derivative in the configuration space \mathcal{M} . The invariance of the weighted 'potential' $\mathbf{L}_{v_I}(\lambda_{\perp}^x V_x) = 0$ is equivalent to (2.29), too. Therefore both of $\lambda_{\perp}^x G_x^{\alpha\beta}$ and $\lambda_{\perp}^x V_x$ are invariant under the diffeomorphism if and only if $\{\lambda_{\perp}^x\}$ satisfies (2.29). What is the meaning of the condition (2.29)? $c_{Ix'}^{(2)x}$ is the 'structure constant' which appears in the algebra (2.5-2.7) and represents a way of transformation of the constraint $\hat{\mathcal{H}}_{\perp x}$ under a group generated by $\{\hat{\mathcal{H}}_I\}$, which corresponds to G . Hence the condition (2.29) means that the coefficient λ_{\perp}^x must change covariantly under the group transformation. In order for the equation (2.29) to be solved consistently, the following integrability condition must be satisfied:

$$c_{IJ}^{(3)K} c_{Kx'}^{(2)x} + c_{Ix'}^{(2)x''} c_{Jx''}^{(2)x} - c_{Jx'}^{(2)x''} c_{Ix''}^{(2)x} + v_I^{\alpha} \partial_{\alpha} c_{Jx'}^{(2)x} - v_J^{\alpha} \partial_{\alpha} c_{Ix'}^{(2)x} = 0 \quad . \quad (2.30)$$

It can be easily confirmed that (2.30) is actually satisfied as a consequence of (2.10) and (2.11) ³. Let $\{\lambda_{\perp x'}^x\}$ be such a complete set of linearly independent real solutions of (2.29) that

$$\lambda_{\perp x'}^x |_{\xi^{\bar{I}} = \xi_0^{\bar{I}}} = \delta_{x'}^x \quad , \quad (2.31)$$

where $\{\xi^{\bar{I}}\}$ is a coordinate system of G and $\{\xi_0^{\bar{I}}\}$ is a set of constants. Then a general positive solution of (2.29) can be written as a linear combination of these solutions:

$$\lambda_{\perp}^x = \bar{\Lambda}_{\perp}^{x'} \lambda_{\perp x'}^x \quad , \quad (2.32)$$

where $\{\bar{\Lambda}_{\perp}^x\}$ is a set of positive functions on the superspace \mathcal{M}/G in the sense that it is a set of positive functions on \mathcal{M} satisfying $\mathbf{L}_{v_I} \bar{\Lambda}_{\perp}^x = 0$. In the remaining of this subsection we restrict λ_{\perp}^x to this form.

From the set of coefficients $\{\lambda_{\perp x'}^x\}$ we can define a set of 'metric' $\{G_x'^{\alpha\beta}\}$ and a set of 'potentials' $\{\bar{V}_x\}$ by

³ The consistency condition (2.30) can be understood as a consequence of Jacobi identities derived from the commutators (2.5-2.7).

$$\begin{aligned}
G'^{\alpha\beta}_x &\equiv \lambda_{\perp x}^{x'} G'^{\alpha\beta}_{x'} \quad , \\
\bar{V}_x &\equiv \lambda_{\perp x}^{x'} V_{x'} \quad .
\end{aligned}
\tag{2.33}$$

Here both of \bar{V}_x and $G'^{\alpha\beta}_x$ are invariant under the group transformation of G . However, since $G'^{\alpha\beta}_x$ may have components in the direction of G , it can not be regarded as a tensor on the superspace \mathcal{M}/G without any modification, while \bar{V}_x can be. We want to modify it in order to regard it as a tensor field on \mathcal{M}/G . For this purpose define the following 'projection operators'.

$$\begin{aligned}
\bar{\mathcal{G}}^{\alpha\beta} &\equiv \mathcal{G}'^{\alpha\beta} - v_I^\alpha \gamma^{IJ} v_J^\beta \quad , \\
\bar{\mathcal{G}}_{\alpha\beta} &\equiv \mathcal{G}'_{\alpha\mu} \bar{\mathcal{G}}^{\mu\nu} \mathcal{G}'_{\nu\beta} \quad , \\
\bar{\mathcal{G}}^\alpha_\beta &\equiv \bar{\mathcal{G}}^{\alpha\nu} \mathcal{G}'_{\nu\beta} \quad , \\
\bar{\mathcal{G}}_\alpha^\beta &\equiv \mathcal{G}'_{\alpha\mu} \bar{\mathcal{G}}^{\mu\beta} \quad ,
\end{aligned}
\tag{2.34}$$

where

$$\begin{aligned}
\mathcal{G}'^{\alpha\beta} &\equiv \bar{\Lambda}_\perp^x G'^{\alpha\beta}_x \quad , \\
\mathcal{G}'_{\alpha\beta} &\equiv \left(\mathcal{G}'^{-1} \right)_{\alpha\beta} \quad , \\
\gamma_{IJ} &\equiv \mathcal{G}'_{\alpha\beta} v_I^\alpha v_J^\beta \quad , \\
\gamma^{IJ} &\equiv \left(\gamma^{-1} \right)^{IJ} \quad .
\end{aligned}
\tag{2.35}$$

It can be easily confirmed by using (2.11) that

$$\mathbb{L}_{v_I} \bar{\mathcal{G}}^{\alpha\beta} = \mathbb{L}_{v_I} \bar{\mathcal{G}}_{\alpha\beta} = \mathbb{L}_{v_I} \bar{\mathcal{G}}^\alpha_\beta = \mathbb{L}_{v_I} \bar{\mathcal{G}}_\alpha^\beta = 0 \quad .
\tag{2.36}$$

Then define the modified tensor $\bar{G}_x^{\alpha\beta}$ by

$$\bar{G}_x^{\alpha\beta} \equiv \bar{\mathcal{G}}^\alpha_\mu G'^{\mu\nu}_x \bar{\mathcal{G}}_\nu^\beta \quad .
\tag{2.37}$$

It is evident that $\bar{G}_x^{\alpha\beta}$ can be regarded as a tensor field on the superspace \mathcal{M}/G since $\mathbb{L}_{v_I} \bar{G}_x^{\alpha\beta} = 0$ and $\bar{G}_x^{\alpha\beta} \bar{\mathcal{G}}_{\beta\gamma} v_I^\gamma = 0$. Note that $\bar{\mathcal{G}}^{\alpha\beta}$ equals to a weighted sum of $\bar{G}_x^{\alpha\beta}$ as

$$\bar{\mathcal{G}}^{\alpha\beta} = \bar{\Lambda}_\perp^x \bar{G}_x^{\alpha\beta} \quad ,
\tag{2.38}$$

and it can also be regarded as a metric tensor of the superspace \mathcal{M}/G . In terms of the metric tensor we can rewrite the (2.22) as the following simultaneous differential equations on \mathcal{M}/G for a real function $W^{(0)}$ on \mathcal{M}/G .

$$\frac{1}{2}\bar{G}_x^{\alpha\beta}\partial_\alpha W^{(0)}\partial_\beta W^{(0)} = \bar{V}_x \quad . \quad (2.39)$$

Our final goal in this subsection is to expand a solution of (2.39) around an *instanton* in the superspace. For this purpose we consider such a congruence of instantons in the superspace that each instanton is an integral curve of the vector field $\bar{\mathcal{N}}^\alpha$ defined by

$$\bar{\mathcal{N}}^\alpha \equiv \bar{\mathcal{G}}^{\alpha\beta}\partial_\beta W^{(0)} \quad , \quad (2.40)$$

and introduce an Euclidean time $\bar{\tau}$ as a coordinate variable in \mathcal{M}/G by

$$\left(\frac{\partial}{\partial\bar{\tau}}\right)^\alpha = \bar{\mathcal{N}}^\alpha \quad . \quad (2.41)$$

If we define a set of vector fields $\{\bar{N}_x^\alpha\}$ by

$$\bar{N}_x^\alpha \equiv \bar{G}_x^{\alpha\beta}\partial_\beta W^{(0)} \quad , \quad (2.42)$$

then

$$\bar{\mathcal{N}}^\alpha = \bar{\Lambda}_\perp^x \bar{N}_x^\alpha \quad , \quad (2.43)$$

and \bar{N}_x^α generates a time reparameterization. Commutators between \bar{N}_x^α and \bar{N}_x^α is zero by using (2.8) and the fact that the covariant derivative \bar{D} is compatible with $\bar{G}_x^{\alpha\beta}$. Hence the set of vector fields $\{\bar{N}_x^\alpha\}$ is integrable in \mathcal{M}/G . Then a coordinate system $\{\bar{\tau}, \eta^{\tilde{x}}, \varphi^{\tilde{n}}\}$ of \mathcal{M}/G can be introduced so that $\{\bar{\tau}, \eta^{\tilde{x}}\}$ is a coordinate system of the integral surface of $\{\bar{N}_x^\alpha\}$. For these definitions, dependence of $W^{(0)}$ on $\{\bar{\tau}, \eta^{\tilde{x}}\}$ is completely determined by (2.39) as

$$\bar{N}_x^\alpha \partial_\alpha W^{(0)} = 2\bar{V}_x \quad . \quad (2.44)$$

Note that there is a one-to-one map from $\{\varphi^{\tilde{n}}\}$ to a space of all integral surface of the set of vector fields $\{\bar{N}_x^\alpha\}$. Moreover $\{\varphi^{\tilde{n}}\}$ is a maximum set of coordinate variables on which

$W^{(0)}$'s dependence is not completely determined by (2.39). Thus we can say that the set of coordinates $\{\varphi^{\bar{n}}\}$ represents all physical degrees of freedom of perturbations around the instanton. The coordinate $\bar{\tau}$ represents an Euclidean time of the congruence of instantons, and $\{\eta^{\tilde{x}}\}$ represents degrees of freedom of time reparameterizations.

Now let us investigate the dependence of $W^{(0)}$ on the physical coordinates $\{\varphi^{\bar{n}}\}$. For this purpose we derive a deviation equation of a vector field in Appendix A. Applying the resulting deviation equation to the vector fields $\bar{\mathcal{N}}^\alpha$ and $\bar{Z}_{\bar{n}}^\alpha \equiv (\partial/\partial\varphi^{\bar{n}})^\alpha$ in the superspace $(\mathcal{M}/G, \bar{\mathcal{G}}_{\alpha\beta})$, we obtain

$$\bar{\mathcal{D}}_{F\perp} \bar{Z}_{\bar{n}}^\alpha = \perp \bar{Z}_{\bar{n}}^\beta \bar{\mathcal{G}}^{\alpha\gamma} \bar{\mathcal{D}}_\gamma \bar{\mathcal{D}}_\beta W^{(0)} \quad , \quad (2.45)$$

$$\bar{\mathcal{D}}_{F\perp}^2 \bar{Z}_{\bar{n}}^\alpha = \perp \bar{Z}_{\bar{n}}^\beta \bar{\mathcal{G}}^{\alpha\gamma} \left(\bar{\mathcal{D}}_\gamma \bar{\mathcal{D}}_\beta \bar{\mathcal{V}} - \bar{\mathcal{R}}_{\gamma\rho\beta\sigma} \bar{\mathcal{N}}^\rho \bar{\mathcal{N}}^\sigma - \frac{3}{2\bar{\mathcal{V}}} \partial_\gamma \bar{\mathcal{V}} \partial_\beta \bar{\mathcal{V}} \right) \quad , \quad (2.46)$$

where $\bar{\mathcal{D}}$ is a covariant derivative compatible with the metric $\bar{\mathcal{G}}_{\alpha\beta}$, $\bar{\mathcal{D}}_F$ denotes a Fermi derivative along $\bar{\mathcal{N}}^\alpha$ made from $\bar{\mathcal{D}}$, and

$$\begin{aligned} \perp \bar{Z}_{\bar{n}}^\alpha &\equiv \bar{Z}_{\bar{n}}^\alpha - \left(\bar{\mathcal{G}}(\bar{\mathcal{N}}, \bar{Z}_{\bar{n}}) / \bar{\mathcal{G}}(\bar{\mathcal{N}}, \bar{\mathcal{N}}) \right) \bar{\mathcal{N}}^\alpha \quad , \\ \perp \bar{\mathcal{G}}^{\alpha\beta} &\equiv \bar{\mathcal{G}}^{\alpha\beta} - \bar{\mathcal{N}}^\alpha \bar{\mathcal{N}}^\beta / \bar{\mathcal{G}}(\bar{\mathcal{N}}, \bar{\mathcal{N}}) \quad , \\ \bar{\mathcal{R}}(X, Y)Z &= \bar{\mathcal{D}}_X \bar{\mathcal{D}}_Y Z - \bar{\mathcal{D}}_Y \bar{\mathcal{D}}_X Z - \bar{\mathcal{D}}_{[X, Y]} Z \quad . \end{aligned} \quad (2.47)$$

The second equation (2.46) makes it possible to investigate how the vector field $\perp \bar{Z}_{\bar{n}}^\alpha$ evolves along a integral line of $\bar{\mathcal{N}}^\alpha$: it corresponds to a linearized equation of motion for the physical degrees of freedom of perturbations around an instanton. With the help of the evolution equation of perturbations, then, the first equation (2.45) says that the second derivative of $W^{(0)}$ in the direction of $\{\perp \bar{Z}_{\bar{n}}^\alpha\}$ is determined as follows.

$$\perp \bar{Z}_{\bar{m}}^\alpha \perp \bar{Z}_{\bar{n}}^\beta \bar{\mathcal{D}}_\alpha \bar{\mathcal{D}}_\beta W^{(0)} = \perp \bar{\mathcal{G}}_{\alpha\beta} \perp \bar{Z}_{\bar{m}}^\alpha \bar{\mathcal{D}}_{F\perp} \bar{Z}_{\bar{n}}^\beta \quad . \quad (2.48)$$

Using these results we can expand a real solution $W^{(0)}$ of the guide equation around an instanton. For this purpose let us introduce a set of new variables $\{\perp \varphi^{\bar{n}}\}$, each of which is an affine length of a geodesic normal to the instanton such that $\perp \varphi^{\bar{n}} = 0$ along Γ and

$$\left(\frac{\partial}{\partial \perp \varphi^{\bar{n}}} \right)^\alpha = \perp \bar{Z}_{\bar{n}}^\alpha \quad \text{along } \Gamma \quad . \quad (2.49)$$

Here Γ denotes the instanton around which we intend to expand a real solution $W^{(0)}$ of the guide equation. Denote the value of $\eta^{\tilde{x}}$ along the instanton Γ by $\eta_0^{\tilde{x}}$. Then we can expand $W^{(0)}$ by ${}_{\perp}\varphi^{\bar{n}}$ around the instanton Γ as follows.

$$W^{(0)}(\bar{\tau}, \eta_0^{\tilde{x}}, {}_{\perp}\varphi^{\bar{n}}) = W^{(0)}(\bar{\tau}, \eta_0^{\tilde{x}}, 0) + \frac{1}{2}\Omega_{\alpha\beta}(\bar{\tau}) {}_{\perp}\bar{Z}_{\bar{m}}^{\alpha} {}_{\perp}\bar{Z}_{\bar{n}}^{\beta} {}_{\perp}\varphi^{\bar{m}} {}_{\perp}\varphi^{\bar{n}} + O({}_{\perp}\varphi^3) \quad , \quad (2.50)$$

where the matrix $\Omega_{\alpha\beta}$ is defined by

$$\Omega_{\alpha\beta}(\bar{\tau}) \equiv {}_{\perp}\bar{\mathcal{G}}_{\alpha\gamma} \left({}_{\perp}\bar{Z}^{-1} \right)_{\beta}^{\bar{n}} \bar{\mathcal{D}}_F {}_{\perp}\bar{Z}_{\bar{n}}^{\gamma} \Big|_{\eta^{\tilde{x}}=\eta_0^{\tilde{x}}, {}_{\perp}\varphi^{\bar{n}}=0} \quad , \quad (2.51)$$

and ${}_{\perp}\bar{Z}_{\bar{n}}^{\alpha}$ follows the evolution equation (2.46). In the expression, $\left({}_{\perp}\bar{Z}^{-1} \right)_{\alpha}^{\bar{n}}$ is defined by

$${}_{\perp}\bar{Z}_{\bar{n}}^{\alpha} \left({}_{\perp}\bar{Z}^{-1} \right)_{\alpha}^{\bar{m}} = \delta_{\bar{n}}^{\bar{m}} \quad , \quad (2.52)$$

and so that $\left({}_{\perp}\bar{Z}^{-1} \right)_{\alpha}^{\bar{n}} {}_{\perp}\bar{Z}_{\bar{n}}^{\beta}$ is a projection operator to a subspace generated by $\{{}_{\perp}\bar{Z}_{\bar{n}}^{\alpha}\}$.

As a result, a general real solution of (2.39) can be expanded as follows.

$$W^{(0)}(\bar{\tau}, \eta_0^{\tilde{x}}, {}_{\perp}\phi^{\alpha}) = W^{(0)}(\bar{\tau}, \eta_0^{\tilde{x}}, 0) + \frac{1}{2}\Omega_{\alpha\beta}(\bar{\tau}) {}_{\perp}\phi^{\alpha} {}_{\perp}\phi^{\beta} + O({}_{\perp}\phi^3) \quad , \quad (2.53)$$

where the new variables $\{{}_{\perp}\phi^{\alpha}\}$ are defined by

$${}_{\perp}\phi^{\alpha} \equiv {}_{\perp}\bar{Z}_{\bar{n}}^{\alpha} {}_{\perp}\varphi^{\bar{n}} \quad , \quad (2.54)$$

provided that its dependence on $\{\bar{\tau}, \eta^{\tilde{x}}\}$ is fixed by (2.44). Note that the set of variables $\{{}_{\perp}\phi^{\alpha}\}$ represents all physical degrees of freedom of perturbations around the instanton. From the expansion (2.53) we obtain the corresponding expansion of the Euclidean wave function.

$$\Psi(\bar{\tau}, \eta_0^{\tilde{x}}, {}_{\perp}\phi^{\alpha}) = \Psi_0(\bar{\tau}) \exp \left[-\frac{1}{\hbar} \left\{ \frac{1}{2}\Omega_{\alpha\beta}(\bar{\tau}) {}_{\perp}\phi^{\alpha} {}_{\perp}\phi^{\beta} + O({}_{\perp}\phi^3, {}_{\perp}\phi^{\alpha}\hbar) \right\} \right] \quad , \quad (2.55)$$

We have expanded a general real solution of (2.39) around an instanton Γ . Finally let us restrict a class of instantons by imposing a specific boundary condition on the expanded wave function. We impose the condition that the matrix Ω is positive definite in the subspace generated by $\{{}_{\perp}\bar{Z}_{\bar{n}}^{\alpha}\}$ along the instanton Γ until a possible turning point, where a *turning*

point means a point on Γ at which $\bar{\mathcal{V}}$ becomes zero. The positivity of the matrix means that probability falls off leaving from the instanton to all physical directions which are orthogonal to the instanton. It seems satisfied if the instanton corresponds to a so-called most probable escape path (MPEP) [3]. In general, if any first class constraints exist for a system, they associate with symmetries of the system and there arises a zero mode problem. However, since by definition Ω is a matrix on a linear space of only physical degrees of freedom, Ω has no zero modes which associate with the first class constraints (2.2). Thus the positivity is necessary for the instanton to have a meaning of a MPEP. Note that we can attach this condition for only a class of instantons, while the expansion (2.53) is possible around an arbitrary instanton. That is because the condition on Ω may restrict behaviors of the corresponding wave function and may violate some physical criterion (eg. asymptotic behavior, etc.). Hereafter we restrict a space of all instantons to the class of those for which we can attach the positivity of Ω , and we adopt the positivity condition. We derive significant results from this condition in the next section.

III. QUANTUM FIELD THEORY OF PERTURBATIONS

In this section let us see how a quantum field theory of perturbations is derived from the Euclidean wave function expanded in the previous section. First we discuss a continuation of the wave function through the turning point. After that we give a field-theoretical interpretation of the wave function.

A. Analytic continuation of the Euclidean wave function

As shown in the previous section, the Euclidean wave function is expanded around an instanton Γ as

$$\Psi(\bar{\tau}, \eta_0^{\tilde{x}}, {}_{\perp}\phi^{\alpha}) = \Psi_0(\bar{\tau}) \exp \left[-\frac{1}{\hbar} \left\{ \frac{1}{2} \Omega_{\alpha\beta}(\bar{\tau}) {}_{\perp}\phi^{\alpha} {}_{\perp}\phi^{\beta} + O\left({}_{\perp}\phi^{\alpha 3}, {}_{\perp}\phi^{\alpha} \hbar\right) \right\} \right] ,$$

where ${}_{\perp}\phi^{\alpha} = 0$ corresponds to the instanton Γ . If we regard ${}_{\perp}\phi^{\alpha}$ as a quantity of order $O(\hbar^p)$ ($0 < p < 1$), then we can neglect the terms of order $O({}_{\perp}\phi^{\alpha 3}, {}_{\perp}\phi^{\alpha}\hbar)$ as a consistent approximation:

$$\Psi(\bar{\tau}, \eta_0^{\tilde{x}}, {}_{\perp}\phi^{\alpha}) = \Psi_0(\bar{\tau}) \exp \left[-\frac{1}{2\hbar} \Omega_{\alpha\beta}(\bar{\tau}) {}_{\perp}\phi^{\alpha} {}_{\perp}\phi^{\beta} \right] . \quad (3.1)$$

It is well known that near a turning point the WKB method is not good. So we need a matching condition of WKB wave functions at the turning point. It is one of the most difficult problems in the WKB approach to the Wheeler-DeWitt equation. The difficulty is mainly due to the following two facts: (1) the configuration space \mathcal{M} is multi-dimensional (infinite dimensional in a continuous limit); (2) the ‘super metric’ $\bar{\mathcal{G}}_{\alpha\beta}$ is not positive definite in the subspace generated by $\{{}_{\perp}\bar{Z}_{\bar{n}}^{\alpha}\}$. Vachaspati and Vilenkin [15] attacked the problem by using a simple model and neglecting the second difficulty. They investigated a Schrodinger equation in a two dimensional configuration space with a positive definite ‘super metric’ and obtain the following matching condition of lowest-order WKB wave functions for the tunneling boundary condition [16]: if the exponent of the wave function is expanded around a classical solution to second order in perturbations orthogonal to it, then the wave function of perturbations is continuous at a turning point. Moreover their result insist that a proper WKB wave function beyond the turning point is obtained by an analytic continuation. As a result the matching problem for their system is equivalent to one for a one-dimensional quantum system. Returning to our system, if the weighted ‘potential’ $\bar{\mathcal{V}}$ does not depend on $\varphi^{\bar{n}}$ near the turning point with an enough accuracy, then the matching problem for our system seems equivalent to one for a one-dimensional quantum system. Hence, in this case, we can expect that their procedure does work. Then we obtain the following form of the wave function beyond the turning point by the analytic continuation with $\bar{\tau} \rightarrow i\bar{t}$.

$$\Psi = \Psi_0(i\bar{t}) \exp \left[-\frac{1}{2\hbar} \Omega_{\alpha\beta}(i\bar{t}) {}_{\perp}\phi^{\alpha} {}_{\perp}\phi^{\beta} \right] , \quad (3.2)$$

where the curve ${}_{\perp}\phi^{\alpha} = 0$ corresponds to that solution of the classical equation of motion (2.18) which is the analytic continuation of the instanton Γ , and $\{{}_{\perp}\phi^{\alpha}\}$ denotes physical perturbations around the classical path. Here the matrix $\Omega_{\alpha\beta}$ can be written as

$$\Omega_{\alpha\beta}(it) = -i {}_{\perp}\bar{\mathcal{G}}_{\alpha\gamma} \left({}_{\perp}\bar{\mathcal{Z}}^{-1} \right)_{\beta}^{\bar{n}} \bar{\mathcal{D}}_F {}_{\perp}\bar{\mathcal{Z}}_{\bar{n}}^{\gamma} \quad , \quad (3.3)$$

where $\bar{\mathcal{D}}_F$ denotes a Fermi-derivative along the analytically-continued classical path and ${}_{\perp}\bar{\mathcal{Z}}_{\bar{n}}^{\alpha}$ is the analytic continuation of ${}_{\perp}\bar{\mathcal{Z}}_{\bar{n}}^{\alpha}$.

B. Field-theoretical interpretation

In quantum field theory, normalized mode functions play a central role. The mode functions are normalized in terms of a so-called Klein-Gordon inner product. Thus, for our discretized system, we propose the following definition of an inner product between two complex vector fields: for complex vector fields X^{α} and Y^{α} on the superspace \mathcal{M}/G ,

$$(X, Y)_{KG} \equiv -i {}_{\perp}\bar{\mathcal{G}}_{\alpha\beta} \left({}_{\perp}X^{\alpha} \bar{\mathcal{D}}_F {}_{\perp}Y^{*\beta} - {}_{\perp}Y^{*\alpha} \bar{\mathcal{D}}_F {}_{\perp}X^{\beta} \right) \quad , \quad (3.4)$$

where $*$ denotes a complex conjugation. For this definition, it can be easily proved that the inner product between $\{{}_{\perp}\bar{\mathcal{Z}}_{\bar{n}}^{\alpha}\}$ is constant along the classical path:

$$\frac{\partial}{\partial t} ({}_{\perp}\bar{\mathcal{Z}}_{\bar{m}}, {}_{\perp}\bar{\mathcal{Z}}_{\bar{n}})_{KG} = 0 \quad . \quad (3.5)$$

We want to normalize the set of vectors $\{{}_{\perp}\bar{\mathcal{Z}}_{\bar{n}}^{\alpha}\}$ since each of them is a solution of perturbed equation of motion and corresponds to the mode function in a continuous limit. By introducing a complex regular matrix $\zeta_k^{\bar{n}}$ whose elements are constant, we take linear combinations of complex conjugates of $\{{}_{\perp}\bar{\mathcal{Z}}_{\bar{n}}^{\alpha}\}$ as

$${}_{\perp}u_k^{\alpha} \equiv \zeta_k^{\bar{n}} {}_{\perp}\bar{\mathcal{Z}}_{\bar{n}}^{*\alpha} \quad . \quad (3.6)$$

Evidently ${}_{\perp}u_k^{\alpha}$ is a candidate for the normalized mode function. By using (3.3) we can express the inner product between $\{{}_{\perp}u_k^{\alpha}\}$ in terms of Ω as

$$({}_{\perp}u_k, {}_{\perp}u_{k'})_{KG} = \left[{}_{\perp}u(\Omega + \Omega^{\dagger}) {}_{\perp}u^{\dagger} \right]_{kk'} \quad . \quad (3.7)$$

The positivity of the Ω in the Euclidean region, which has been introduced in the last paragraph of subsection IIB, means that the matrix $({}_{\perp}u_k, {}_{\perp}u_{k'})_{KG}$ calculated in (3.7) is

positive definite at the turning point. Hence, due to the constancy of the matrix elements implied by (3.5), we can choose the transformation matrix $\zeta_k^{\bar{n}}$ so that

$$(\perp u_k, \perp u_{k'})_{KG} = \delta_{kk'} \quad (3.8)$$

along the analytically-continued classical path. Then $\{\perp u_k^\alpha\}$ is specified as a set of orthonormalized solutions of the simultaneous equations

$$\bar{\mathcal{D}}_{F\perp}^2 u_k^\alpha = -\perp u_k^\beta \bar{\mathcal{G}}^{\alpha\gamma} \left(\bar{\mathcal{D}}_\gamma \bar{\mathcal{D}}_\beta \bar{\mathcal{V}} + \bar{\mathcal{R}}_{\gamma\rho\beta\sigma} \left(\frac{\partial}{\partial t} \right)^\rho \left(\frac{\partial}{\partial t} \right)^\sigma - \frac{3}{2\bar{\mathcal{V}}} \partial_\gamma \bar{\mathcal{V}} \partial_\beta \bar{\mathcal{V}} \right) , \quad (3.9)$$

$$0 = \bar{\mathcal{G}}_{\alpha\beta} \bar{\mathcal{N}}^\alpha \perp u_k^\beta , \quad (3.10)$$

and is a complete set of basis vectors in a subspace generated by $\{(\partial/\partial \perp \varphi^{\bar{n}})^\alpha\}$. Equation (3.9-3.10) is a linearized evolution equation of orthogonal perturbations around the analytically-continued classical path. The boundary condition of $\perp u_k^\alpha$ is that the matrix $\Omega_{\alpha\beta}$ defined by

$$\Omega_{\alpha\beta} \equiv -i \perp \bar{\mathcal{G}}_{\alpha\gamma} (\perp u^{-1})_\beta^{*k} \bar{\mathcal{D}}_{F\perp} u_k^{*\gamma} \quad (3.11)$$

is real and positive definite in the subspace generated by $\{\perp \bar{Z}_{\bar{n}}^\alpha\}$ when it is analytically-continued back to the Euclidean region (see arguments in the last paragraph of subsection IIB), where $*$ denotes a complex conjugation and $(\perp u^{-1})_\alpha^k$ is defined by

$$\perp u_k^\alpha (\perp u^{-1})_\alpha^{k'} = \delta_k^{k'} , \quad (3.12)$$

and so that $(\perp u^{-1})_{\alpha\perp}^k u_k^\beta$ is a projection operator to the subspace generated by $\{(\partial/\partial \perp \varphi^{\bar{n}})^\alpha\}$.

Finally we show that the wave function (3.2) can actually be interpreted in terms of a quantum-mechanical system, which will become a quantum field theory in a continuous limit. Consider a quantum-mechanical system described by the hamiltonian

$$\hat{H} = \frac{1}{2} \left(\perp \bar{\mathcal{G}}^{\alpha\beta} \perp \hat{\pi}_\alpha \perp \hat{\pi}_\beta + \perp V_{\alpha\beta} \perp \hat{\phi}^\alpha \perp \hat{\phi}^\beta \right) , \quad (3.13)$$

and the equation of motion

$$i\hbar \bar{\mathcal{D}}_F \hat{O} = [\hat{O}, \hat{H}] \quad (3.14)$$

for an arbitrary operator \hat{O} , where ${}_{\perp}\hat{\pi}_{\alpha}$ is a momentum conjugate to ${}_{\perp}\hat{\phi}^{\alpha}$ and

$${}_{\perp}V_{\alpha\beta} \equiv {}_{\perp}\bar{\mathcal{G}}_{\alpha}^{\mu} {}_{\perp}\bar{\mathcal{G}}_{\beta}^{\nu} \left(\bar{\mathcal{D}}_{\mu} \bar{\mathcal{D}}_{\nu} \bar{\mathcal{V}} + \bar{\mathcal{R}}_{\mu\rho\nu\sigma} \left(\frac{\partial}{\partial \bar{t}} \right)^{\rho} \left(\frac{\partial}{\partial \bar{t}} \right)^{\sigma} - \frac{3}{2\bar{\mathcal{V}}} \partial_{\mu} \bar{\mathcal{V}} \partial_{\nu} \bar{\mathcal{V}} \right) . \quad (3.15)$$

The equation of motion says that the operator ${}_{\perp}\hat{\phi}^{\alpha}$ can be expanded as follows.

$${}_{\perp}\hat{\phi}^{\alpha} = \hbar^{1/2} \sum_k \left(\hat{a}_k {}_{\perp}u_k^{\alpha} + \hat{a}_k^{\dagger} {}_{\perp}u_k^{*\alpha} \right) , \quad (3.16)$$

where \hat{a}_k and \hat{a}_k^{\dagger} are annihilation and creation operators which satisfy

$$\begin{aligned} [\hat{a}_k, \hat{a}_{k'}^{\dagger}] &= \delta_{kk'} , \\ [\hat{a}_k, \hat{a}_{k'}] &= 0 , \\ [\hat{a}_k^{\dagger}, \hat{a}_{k'}^{\dagger}] &= 0 . \end{aligned} \quad (3.17)$$

For the representation (3.16) with the normalization (3.8), the annihilation operator \hat{a}_k can be expressed as a linear combination of terms of the form

$${}_{\perp}\hat{\pi}_{\alpha} - i\Omega_{\alpha\beta} {}_{\perp}\hat{\phi}^{\beta} . \quad (3.18)$$

Hence the wave function (3.2) is annihilated by all the annihilation operators:

$$\hat{a}_k \Psi = 0 \quad (3.19)$$

for all k . This says that the wave function (3.2) represents the vacuum state determined by the vectors $\{{}_{\perp}u_k^{\alpha}\}$. Thus, in a continuous limit, the corresponding wave function can be interpreted in terms of a quantum field theory: it represents the vacuum state determined by that set of positive-frequency mode functions which is a continuous limit of $\{{}_{\perp}u_k^{\alpha}\}$.

C. Reduced Lagrangian

When we intend to apply the formalism developed in this paper, what we have to do is: (1) to fix a MPEP; (2) to give a coordinate system in a space of all gauge-invariant perturbations (physical perturbations) around the MPEP; (3) to seek explicit form of the

effective Hamiltonian (3.13) and the matrix (3.11); (4) to calculate any wanted quantities by using the quantum field theory described by the effective Hamiltonian. The positive-frequency mode function is defined so that the matrix (3.11) is real and positive definite in the subspace spanned by the gauge-invariant perturbations in the Euclidean region.

In this subsection we show that the effective Hamiltonian and the matrix can be obtained from an reduced Lagrangean of the gauge-invariant perturbations. Thus the reduced Lagrangean gives a convenient prescription to perform the procedure (3).

The classical Hamiltonian (2.15) corresponds to the Lagrangian

$$\begin{aligned} L &= p_\alpha \dot{q}^\alpha - H \\ &= \frac{1}{2} \mathcal{G}_{\alpha\beta} (\dot{q}^\alpha - \mathbf{v}^\alpha) (\dot{q}^\beta - \mathbf{v}^\beta) - \mathcal{V} \quad . \end{aligned} \quad (3.20)$$

This Lagrangian describes a dynamics in the full configuration space \mathcal{M} . When we go to the superspace \mathcal{M}/G , the corresponding reduced Lagrangian \bar{L} in \mathcal{M}/G is

$$\bar{L} = \frac{1}{2} \bar{\mathcal{G}}_{\alpha\beta} \dot{\bar{q}}^\alpha \dot{\bar{q}}^\beta - \bar{\mathcal{V}} \quad , \quad (3.21)$$

where $\{\bar{q}^\alpha\}$ is a coordinate system in \mathcal{M}/G . The kinetic part for the gauge-invariant perturbations is ⁴

$$\frac{1}{2} {}_\perp \bar{\mathcal{G}}_{\alpha\beta} \dot{\phi}^\alpha {}_\perp \dot{\phi}^\beta \quad (\in \bar{L}) \quad . \quad (3.22)$$

Thus the form of ${}_\perp \bar{\mathcal{G}}_{\alpha\beta}$, which is necessary and sufficient to obtain the explicit form of the matrix (3.11), can be read from the kinetic part of the reduced Lagrangian.

When the dot $\dot{}$ is identified with $\bar{\mathcal{D}}_F$, \bar{L} must derive the field equation (3.9) with ${}_\perp u_k^\alpha$ replaced by ${}_\perp \phi^\alpha$ since by definition the field equation (3.9) is nothing else the evolution equation of the orthogonal perturbations. From this fact, the quadratic part of the reduced Lagrangian must be of the form

$$\frac{1}{2} {}_\perp \bar{\mathcal{G}}_{\alpha\beta} \dot{\phi}^\alpha {}_\perp \dot{\phi}^\beta - \frac{1}{2} {}_\perp V_{\alpha\beta} \phi^\alpha {}_\perp \phi^\beta \quad (\in \bar{L}) \quad , \quad (3.23)$$

⁴ Note that terms linear in ${}_\perp \phi^\alpha$ is automatically zero because of the background equation.

where $\perp V_{\alpha\beta}$ is defined by (3.15). The effective Hamiltonian (3.13) can be obtained from the effective action by usual prescription. Thus, in a continuous limit, the reduced Lagrangian leads us to the effective quantum field theory considered in the previous subsection.

After all, when we intend to investigate a state of the physical perturbations after a quantum tunneling, what we have to do is: (a) to fix a MPEP; (b) to give a coordinate system in a space of all gauge-invariant perturbations around the MPEP; (c) to obtain the reduced Lagrangian of the gauge-invariant perturbations by simply reducing the original Lagrangian to the space of all gauge-invariant perturbations; (d) to use the quantum field theory of the perturbations constructed from the reduced Lagrangian. From the effective action we can read off the form of the matrix (3.11), which must be real and positive in the subspace generated by gauge-invariant perturbations in the Euclidean region.

IV. SUMMARY AND DISCUSSION

Throughout this paper, we have investigated the simultaneous differential equations (2.2) with the algebra (2.5-2.7) by using the WKB method. We have mentioned that, in a continuous limit, (2.2) and (2.5-2.7) become generalizations of the Wheeler-DeWitt equation (1.8) and the Dirac algebra (1.9-1.11), respectively. We have defined an Euclidean wave function as that solution of the equations (2.2) whose lowest-order part in the WKB expansion (2.21) is real in a region of the configuration space. We have called the region an Euclidean region. The lowest-order part in \hbar has been expanded in the superspace around an instanton, by using a deviation equation of a vector field tangent to a congruence of instantons. The instanton around which we expand the wave function corresponds to a so-called most probable escape path (MPEP). Then we have shown that, when the expanded wave function is analytically-continued beyond the Euclidean region, the continued wave function can be understood in terms of a quantum-mechanical system, which will become a quantum field theory in a continuous limit. The corresponding state of physical perturbations around the analytically-continued classical path is the vacuum state determined by 'positive-frequency

mode functions' $\{\perp u_k^\alpha\}$. Each 'mode function' must be normalized with respect to the norm (3.4) and satisfy the evolution equations (3.9-3.10), which correspond to linearized classical equations of motion for perturbations orthogonal to the analytically-continued classical path. The positive-frequency mode functions must satisfy also the following boundary condition: the matrix $\Omega_{\alpha\beta}$ defined by (3.11) must be real and positive definite in the subspace generated by $\{\perp \bar{Z}_{\bar{n}}^\alpha\}$ when it is analytically-continued back to the Euclidean region. Note that the boundary condition of the mode functions is dependent of the matching condition of the WKB wave functions. In subsection III A, in the case that the weighted 'potential' $\bar{\mathcal{V}}$ does not depend on $\varphi^{\bar{n}}$ near the turning point with an enough accuracy, we have adopted such a matching condition that the WKB wave functions are analytic continuations of each other. If a future analysis for more general case gives a different matching condition of the WKB wave functions, then the mode functions must be matched with those in the Euclidean region in a different way and the corresponding matrix $\Omega_{\alpha\beta}$ must be real and positive definite in the subspace generated by $\{\perp \bar{Z}_{\bar{n}}^\alpha\}$. Anyway, the state of perturbations after a quantum tunneling is determined uniquely by the positive-frequency mode functions. Thus a quantum field theory is effective to investigate a state of physical perturbations after a quantum tunneling. We have shown that the effective Lagrangian describing the field theory is obtained by simply reducing the original Lagrangian to the subspace spanned by the physical perturbations around the MPEP.

The result of this paper does not depend on the operator ordering since we have concentrated only on (2.8-2.11) and (2.22), which are independent of the operator ordering. Moreover the result can be applied to a general background, which corresponds to a most probable escape path (MPEP). Thus, for a general MPEP, a quantum field theory is effective to investigate a state of all physical perturbations after a quantum tunneling with gravity. Due to the result of this paper we can safely use the quantum field theory to obtain a CMB anisotropy, a spectrum of primordial gravitational waves, etc. in some inflationary scenarios. For example, in the one-bubble inflationary scenario, Tanaka and Sasaki [10] calculated the reduced Lagrangian and quantized the gravitational perturbations. Although

they did not justify their treatment from the point of view of the Wheeler-DeWitt equation, the result of this paper do justify it without writing down the explicit form of the perturbed Wheeler-DeWitt equation.

Finally we propose a possible definition of a two point function to strengthen the field theoretical interpretation. For a function f on \mathcal{M}/G , define a one-parameter family of expectation values $\langle f \rangle_{\bar{t}}$ along the analytically-continued classical path.

$$\langle f \rangle_{\bar{t}} \equiv \left[\frac{f(\Pi_{\bar{n}} d_{\perp} \varphi^{\bar{n}}) \sqrt{|\det'_{\perp} \bar{\mathcal{G}}|} |\Psi|^2}{f(\Pi_{\bar{n}} d_{\perp} \varphi^{\bar{n}}) \sqrt{|\det'_{\perp} \bar{\mathcal{G}}|} |\Psi|^2} \right]_{\bar{t}, \eta^{\hat{x}} = \eta_0^{\hat{x}}}, \quad (4.1)$$

where \det' represents a determinant of the followed matrix restricted to the subspace generated by $\{(\partial/\partial_{\perp} \varphi^{\bar{n}})^{\alpha}\}$. It is evident that for the wave function (3.2) the expectation value of $_{\perp} \phi^{\alpha}$ is zero: $\langle _{\perp} \phi^{\alpha} \rangle_{\bar{t}} = 0$. On the other hand the expectation value of $\{_{\perp} \phi^{\alpha}, _{\perp} \phi^{\beta}\}$ is not zero:

$$\langle \{_{\perp} \phi^{\alpha}, _{\perp} \phi^{\beta}\} \rangle_{\bar{t}} \approx \hbar \left[(\Omega + \Omega^{\dagger})^{-1} \right]^{\alpha\beta} + \hbar \left[(\Omega + \Omega^{\dagger})^{-1} \right]^{\beta\alpha}, \quad (4.2)$$

where the right hand side is calculated on the classical path. Note that ' \approx ' means that the difference between the right and the left hand side is sufficiently small when the wave function is so peaked along the classical path that an orthogonal transformation of the hermite part of Ω and the evaluation of the expectation value is approximately commutable. From (3.7) we can show that

$$(\Omega + \Omega^{\dagger})^{-1} = _{\perp} u^{\dagger} (_{\perp} u, _{\perp} u)_{KG \perp u}^{-1} _{\perp} u, \quad (4.3)$$

and

$$\langle \{_{\perp} \phi^{\alpha}, _{\perp} \phi^{\beta}\} \rangle_{\bar{t}} \approx \hbar \left[_{\perp} u^{\dagger} (_{\perp} u, _{\perp} u)_{KG \perp u}^{-1} \right]^{\alpha\beta} + \hbar \left[_{\perp} u^{\dagger} (_{\perp} u, _{\perp} u)_{KG \perp u}^{-1} \right]^{\beta\alpha}. \quad (4.4)$$

In subsection III B we have shown that the matrix $\zeta_k^{\bar{n}}$ can be chosen so that $(_{\perp} u_k, _{\perp} u_{k'})_{KG} = \delta_{kk'}$. For this choice of $\zeta_k^{\bar{n}}$, the two point function is reduced to

$$\langle \{_{\perp} \phi^{\alpha}, _{\perp} \phi^{\beta}\} \rangle_{\bar{t}} \approx \hbar \sum_k \left(_{\perp} u_k^{*\alpha} _{\perp} u_k^{\beta} + _{\perp} u_k^{\alpha} _{\perp} u_k^{*\beta} \right). \quad (4.5)$$

The right hand side of (4.5) can be understood as a two point function of the quantum-mechanical system introduced in subsection III B as follows. The representation (3.16) leads the two point function

$$\langle 0 | \{ \hat{\phi}^\alpha, \hat{\phi}^\beta \} | 0 \rangle = \hbar \sum_k \left(\hat{u}_k^{*\alpha} \hat{u}_k^\beta + \hat{u}_k^\alpha \hat{u}_k^{*\beta} \right) , \quad (4.6)$$

where the state $|0\rangle$ is the vacuum state of the quantum-mechanical system:

$$\hat{a}_k |0\rangle = 0 \quad (4.7)$$

for $\forall k$. The two point function (4.6) is the right hand side of (4.5) itself. Thus the two point function based on the wave function (3.2) is equivalent to that based on the quantum-mechanical system.

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APPENDIX A: DEVIATION EQUATION OF A VECTOR FIELD

In this appendix we derive a deviation equation for a spacelike (and timelike) vector field in a pseudo-Riemannian manifold. By spacelike (timelike) we mean that its norm with respect to the metric is positive (negative).

Let $(M, g_{\alpha\beta})$ be a pseudo-Riemannian manifold and N^α be a vector field on M . It is assumed that $g_{\alpha\beta} N^\alpha N^\beta \neq 0$ in a region $E (\in M)$. Then introduce another vector field Z^α in E such that

$$[N, Z]^\alpha = 0 . \quad (A1)$$

Locally this condition is a necessary and sufficient condition in order for N^α and Z^α to define independent coordinate variables by

$$\begin{aligned}\left(\frac{\partial}{\partial\tau}\right)^\alpha &= N^\alpha \quad , \\ \left(\frac{\partial}{\partial\lambda}\right)^\alpha &= Z^\alpha \quad .\end{aligned}\tag{A2}$$

When we apply equations derived in this appendix to the system investigated in subsection IIB, N^α corresponds to a gradient vector field (2.40) of a real solution of (2.39). Then λ corresponds to a coordinate $\varphi^{\bar{n}}$ distinguishing physically different integral lines of N^α . The condition (A1) can be written explicitly as

$$\dot{Z}^\alpha = Z^\beta \nabla_\beta N^\alpha \quad ,\tag{A3}$$

where ∇ is a covariant derivative compatible with $g_{\alpha\beta}$ and

$$\dot{X}^\alpha \equiv N^\beta \nabla_\beta X^\alpha\tag{A4}$$

for $X^\alpha \in TM$. Operating $N^\beta \nabla_\beta$ to it, we obtain the following equation.

$$\ddot{Z}^\alpha = Z^\beta \nabla_\beta \dot{N}^\alpha - R^\alpha_{\rho\beta\sigma} N^\rho N^\sigma Z^\beta \quad ,\tag{A5}$$

where $R(X, Y)Z = \nabla_X \nabla_Y Z - \nabla_Y \nabla_X Z - \nabla_{[X, Y]} Z$.

Without any modification, these equations are not effective enough to investigate the dependence of the wave function on the perturbation around the instanton. It is because Z^α may have a component in the direction of N^α . We want to modify these equations so that Z^α appears as a vector projected to a hypersurface orthogonal to N^α in the modified equations. To make the modification possible, we define a so-called Fermi transported basis. First a Fermi derivative ∇_F along N^α is defined by

$$\nabla_F X^\alpha = \dot{X}^\alpha + \frac{1}{N^2} g(X, \dot{N}) N^\alpha - \frac{1}{N^2} g(X, N) \dot{N}^\alpha\tag{A6}$$

for $X^\alpha \in TM$, where $N^2 \equiv g(N, N)$ [17]. Note that N^2 is positive (negative) when N^α is spacelike (timelike). The Fermi derivative has the following well known properties.

1. If N^α is tangent to a congruence of geodesics, then $\nabla_F X^\alpha = \dot{X}^\alpha$ for $X^\alpha \in TM$.

2. $\nabla_F \perp X^\alpha = \perp(\perp X)^\alpha$ for $X^\alpha \in TM$, where $\perp X^\alpha \equiv X^\alpha - N^\alpha g(N, X)/N^2$.

3. $\nabla_F(N^\alpha/\sqrt{|N^2|}) = 0$.

4. If X^α and Y^α ($\in TM$) satisfy $\nabla_F X^\alpha = \nabla_F Y^\alpha = 0$, then $N^\alpha \partial_\alpha g(X, Y) = 0$.

The first and the second properties suggest that the Fermi derivative is effective for our purpose. From the third and the forth properties, along an integral curve of N^α the Fermi transported basis can be defined by

$$\begin{aligned} e_0^\alpha &\equiv N^\alpha / \sqrt{|N^2|} \ , \\ \nabla_F e_i^\alpha &= 0 \ , \end{aligned} \tag{A7}$$

and so that $g(e_0, e_i) = 0$ and $g(e_i, e_j) = \eta_{ij}$, where η_{ij} is a constant regular matrix. By using this basis, we can expand Z^α as

$$Z^\alpha = Z^0 e_0^\alpha + Z^i e_i^\alpha \ , \tag{A8}$$

where $Z^0 \equiv g(Z, e_0)/g(e_0, e_0)$ and $Z^i \equiv (\eta^{-1})^{ij} g(Z, e_j)$. Substituting (A8) into (A3) and (A5), we can show by explicit calculation that

$$\begin{aligned} \frac{\partial}{\partial \tau} Z^i &= Z^j \nabla_j N^i \ , \\ \frac{\partial^2}{\partial \tau^2} Z^i &= Z^j \nabla_j \dot{N}^i - R^i_{\rho j \sigma} N^\rho N^\sigma Z^j - \frac{1}{N^2} \dot{N}^i Z^j \eta_{jk} \dot{N}^k - \frac{1}{N^2} \dot{N}^i Z^j \nabla_j N^2 \ , \end{aligned} \tag{A9}$$

where

$$\begin{aligned} \dot{N}^i &\equiv (\eta^{-1})^{ik} g_{\alpha\beta} e_k^\alpha \dot{N}^\beta \ , \\ \nabla_j N^i &\equiv (\eta^{-1})^{ik} g_{\alpha\beta} e_k^\alpha e_j^\gamma \nabla_\gamma N^\beta \ , \\ \nabla_j \dot{N}^i &\equiv (\eta^{-1})^{ik} g_{\alpha\beta} e_k^\alpha e_j^\gamma \nabla_\gamma \dot{N}^\beta \ , \\ R^i_{\rho j \sigma} &\equiv (\eta^{-1})^{ik} g_{\alpha\beta} e_k^\alpha e_j^\gamma R^\beta_{\rho \gamma \sigma} \ , \\ \nabla_j (N^2) &\equiv e_j^\alpha \partial_\alpha (N^2) \ . \end{aligned} \tag{A10}$$

The result can be rewritten as a covariant form:

$$\begin{aligned}
\nabla_F \perp Z^\alpha &= \perp Z^\beta \perp (\nabla_\beta N)^\alpha \quad , \\
\nabla_F^2 \perp Z^\alpha &= \perp Z^\beta \perp (\nabla_\beta \dot{N})^\alpha - \perp R^\alpha_{\rho\beta\sigma} N^\rho N^\sigma \perp Z^\beta - \frac{1}{N^2} g(\perp Z, \dot{N}) \perp \dot{N}^\alpha \\
&\quad - \frac{1}{N^2} g(\perp Z, \nabla N^2) \perp \dot{N}^\alpha \quad ,
\end{aligned} \tag{A11}$$

where

$$\begin{aligned}
\perp (\nabla_\beta N)^\alpha &\equiv \nabla_\beta N^\alpha - \frac{N^\alpha N_\gamma}{N^2} \nabla_\beta N^\gamma \quad , \\
\perp (\nabla_\beta \dot{N})^\alpha &\equiv \nabla_\beta \dot{N}^\alpha - \frac{N^\alpha N_\gamma}{N^2} \nabla_\beta \dot{N}^\gamma \quad , \\
\perp R^\alpha_{\rho\beta\sigma} &\equiv R^\alpha_{\rho\beta\sigma} - \frac{N^\alpha N_\gamma}{N^2} R^\gamma_{\rho\beta\sigma} \quad .
\end{aligned} \tag{A12}$$

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